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Application Note AN-0210-1

Tracking Instrument Behavior

A frequently asked question is "How can I be sure that my instrument is performing normally?" Before we can answer this question we must define what is normal. Nothing is absolute in radiation measurements. Any such measurement, repeated under supposedly identical conditions, will yield a variety of answers. Determining if an instrument is behaving normally is therefore a statistical problem. Are the variances of the measurements truly behaving like a statistical model would predict? This begs the question, "What is the model?"

There is no avoiding it, a working knowledge of statistics is necessary to understand the limitations of radiation counters, predict the associated errors, and determine if an instrument is performing as it should. This knowledge also helps explain the need for many of the features built into most contemporary instruments. For our purposes the emphasis is on "a working knowledge". You do not need to know how the statistical tools were derived, only when how to apply them.

The Goal

Before your eyes start to roll back in your head remember the lead question. The following several paragraphs present a synopsis of the statistical concepts related to radiation counting. An excellent overview of statistics as it relates to radiation applications is presented in Knoll's book "Radiation Detection and Measurement". Knoll states that one of the applications "… involves the use of these statistics to determine whether a set of multiple measurements of the same physical quantity shows an amount of internal fluctuation that is consistent with statistical predictions. This application usually is used to determine whether a particular counting system is functioning normally." The goal of this document is to explain that stated application in conjunction with alpha/beta counters.

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Statistical Models

A collection of radiation measurements (repeated under supposedly identical conditions) yields a distribution. A variety of distribution models have been developed describing statistical behavior. The grandfather of them all is the binomial distribution but it is cumbersome to use in its standard form. More simplified derivatives apply to the specific characteristics related to the radiation decay processes. One such derivative is the Poisson distribution.

The Poisson Distribution

Siméon-Denis Poisson was a French mathematician. His work focused on certain random variables that count, among other things, the number of discrete occurrences that take place during a time interval of given length – sounds like a radiation counter doesn't it.

The key word above is <u>discrete</u> as in the decay of an atom where either a particle is ejected or not, a fractional particle does not exist, and with an instrument the particle is either detected or not. The Poisson distribution applies to observations with a large number of possible events, each of which is rare; conditions that again describe the nuclear decay of atoms.

The Poisson distribution equation is described in a number of texts and on-line sources so it won't be repeated here. While the Poisson distribution is much easier to work with than its parent, the binomial, when the mean value is low it lacks the symmetry that would make it even easier to use. The symmetry we speak of is a characteristic of a "normal distribution".

Unlike the Poisson distribution, the measured variables in a normal distribution can take on a continuum of values. With a radiation detection measurement that is not the case, an event either occurs or it does not. None-the-less, under certain conditions the normal distribution may be used.

If the mean value of a Poisson distribution is large (e.g., greater than 20) the equation describing the distribution becomes almost identical to that of the normal distribution.

Normal Distribution

The normal distribution, sometimes called a Gaussian distribution, with its familiar bell curve describes measurements that take on a continuum of values. The normal distribution (Figure 1) has a mean value μ representing the true value of the quantity *x* being measured. The curve is further defined by σ the standard deviation.

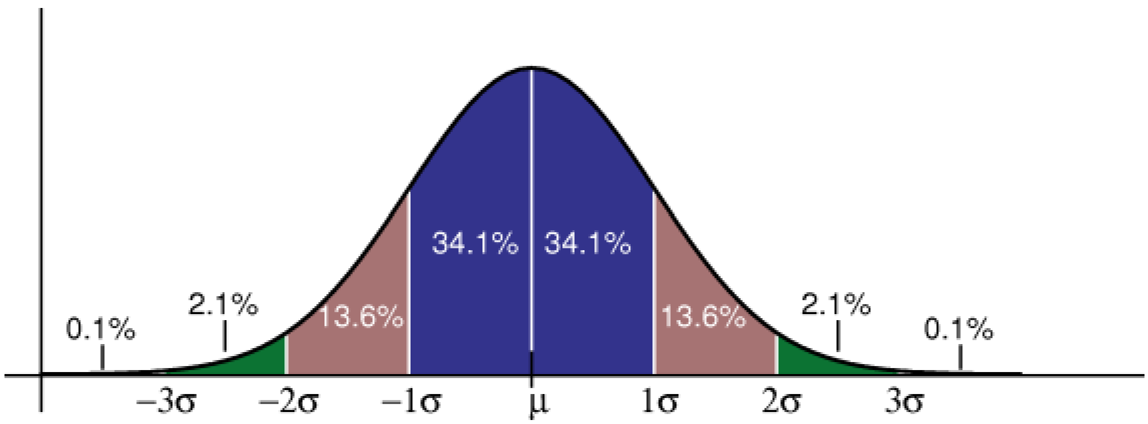

Figure 1 – Normal Distribution

A normal distribution containing *N* observations has a mean value μ defined as:

$$\mu \equiv \bar{x}$$

Mean Value - Equation 1

In addition it has a standard deviation σ defined as:

$$\sigma \equiv \sqrt{\bar{x}}$$

Standard Deviation - Equation 2

As the figure above illustrates, 68% of the values lie within 1 standard deviation of the mean; 95.4% lie within 2 standard deviations; and 99.6% lie within 3 standard deviations.

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With a set of experimental data:

$$\overline{x_e} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Experimental Mean – Equation 3

And the experimental standard deviation is,

$$\sigma_e = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x_e})^2}{N-1}}$$

Experimental Std Deviation – Equation 4

That is our entire synopsis of statistics; let's return the focus back to the goal. Knowing that radiation counters are expected to behave according to the statistical models it follows that we need a quantitative method of comparing actual behavior with predicted behavior. One such method is the chi square test.

Chi-Square Test

The chi squared test is a quantitative way of judging whether or not the variances of a population of counting results match statistical predictions to some stated degree of certainty. Used in this way it is called a "goodness of fit" test.

The chi squared value is represented by X^2. The value *N* is the number of trials, observations or count intervals. The value of a specific observation *i* is x_i. The mean of the population of observations is $\overline{x_e}$ as defined in Equation 3 above. With this information the chi squared may be calculated using Equation 5.

$$X^2 \equiv \frac{\sum_{i=1}^{N}(x_i - \overline{x_e})^2}{\overline{x_e}}$$

Chi Squared – Equation 5

The relationship between the actual or experimental variance and the predicted variance is given by:

$$X^2 = (N-1)\frac{Actual\,Variance}{Predicted\,Variance}$$

Chi Squared ∝Variances – Equation 6

In this expression *(N-1)* is equivalent to a statistical value referred to as "the number of degrees of freedom". If the actual variance perfectly matches the predicted variance the chi squared value equals the number of degrees of freedom.

If the chi squared value is greater than the number of degrees of freedom then there is greater variance than expected and vice versa. From an analytical perspective the former would indicate that the instrument is under the influence of some abnormal fluctuations. Less variance than expected might indicate the instrument is being influenced by some periodic oscillation such as an *ac* noise source (i.e., the population is not random).

Chi squared is a probability distribution with finite boundaries. The general test for a "goodness of fit" for counting instruments is a 99% confidence level. For example we can state that if a calculated chi squared value for a set of experimental data is less than some upper critical value and greater than some lower critical value there is a 99% probability that the experimental variances agree with the model.

To find the critical value boundaries for specific probabilities and degrees of freedom we turn to chi square tables or use a chi square calculator. Either way, we use the number of degrees of freedom, the probabilities, and then extract the critical values – the specific probabilities in our case are 0.99 for the upper critical value and 0.01 for the lower critical value. The table below shows the critical values used with a chi squared test for various sets of counts and a 99% confidence level.

N	Lower C.V.	Upper C.V.
10	2.09	21.67
20	7.63	36.19
50	28.94	74.92
100	69.23	134.64
150	111.80	192.07
200	155.55	248.33

The chi square test is only valid when a normal distribution is assumed therefore a minimum of 10 observations (and preferably 20) are needed with a minimum of 10 counts per observation – adjust the counting time accordingly.

Trend Charts

The chi squared test is generally applied to a set of data taken in short succession. It has customarily been used as a daily or weekly check on an instrument as the benchmark for suitability of use. In these checks ten or twenty short counts are made on a radioactive source and the chi squared values calculated and compared to the expected range. When a problem is suspected, the chi squared test and a graphic display of data are valuable diagnostic tools.

Nothing replaces the graph as an intuitive tool for judging instrument performance and recognizing problem patterns. One such use is multiple short term backgrounds to spot abnormal bursts such as a detector breakdown. Most of Protean's instruments provide a trending function as a diagnostics tool. Individual data points are plotted about the mean of the population with the ±2 and ±3 standard deviation lines shown to aid in the analysis. This function also calculates chi squared values for the data set.

As previously discussed the chi squared value provides a quantitative measure of the "goodness of fit". The graphical presentation helps identify and draw conclusions about short term counting trends that might be associated with changes in the count room environment or equipment malfunctions.

Control Charts

The chi squared test with or without Protean's trending and charting accessories are intended as a short term test. Long term tracking of instrument performance is relegated to what Protean refers to as the control chart. Semantically this name may not be correct since these charts do not exert any direct control over the instrument but the name and its described usage is commonly accepted. What follows is a discussion relating to Protean's interpretation and use of these charts.

Protean's control charts consist of two data sets. The first set establishes the limits for acceptable or expected performance; hence our referral to it is the "Limits Data". Since this data set is expected to represent a Poisson distribution simplified as a normal distribution, the mean value of the population should be greater than 20 and there should be at least 10 (preferably more) observations (count intervals) in the set. The counting time for each interval should be chosen to meet the ≥20 count criterion. This may not be practical when creating background control charts for systems exhibiting very low inherent backgrounds, *in which case remember that the normal distribution rules are not valid*.

The second data set contains the post historical data, i.e. the routine checks; hence we refer to it as the "Checks Data". The normal distribution predicts that 95.4% of the checks should lie within 2 standard deviations as determined from the Limits Data; and 99.6% should lie within 3 standard deviations. Outliers would indicate that the checks are influenced by something other than counting statistics.

A frequently asked question is why, when some Limits Data is replicated onto a spread sheet and the standard deviations calculated, are they different from those reported by the instrument? The likely answer is propagation of errors.

The spread sheet functions only calculate the error associated with the statistics of the count. To quote from a circa 1960 document long lost in antiquity, "… Generally speaking, the stability of counting apparatus is such that the fundamental statistical error will dominate if it is greater than 1 per cent." In this quote, what the statistical error dominates is the *systematic error* associated with the counting apparatus.

Using Equation 7 a statistical error of 1% is equivalent to an accumulated count of 10,000 counts. In low level counting situations this is seldom the case; however, with check sources it is easy to exceed 10,000 counts even during a 1 minute count!

$$e_s = \frac{\sqrt{\bar{x}}}{\bar{x}}$$

Fractional Statistical Error – Equation 7

The systematic error represents any variance associated with the counting apparatus and may be a combination of barometric and temperature variances, gas quality for proportional counters, electronic variances, and sample positioning variances.

Positioning variances are a combination of a non-uniform distribution of radioactivity on the surface of the check source and any non-uniformity in the efficiency across the active area of the detector. Seldom does one ensure that the check source is placed in exactly the same orientation between each check.

Real life experiences are the best teachers. An example of systematic error was encountered when a number of associated laboratories complained that their counting systems frequently failed the 2σ test in their self-defined control charts. Their imposed rules were to collect >100,000 counts per daily check. From Equation 7 the statistical error in this case is <0.3% and their control chart limits <±0.6% ($\pm 2\sigma$). The local weather service confirmed that during this same period the barometric pressure of the atmosphere varied by over 10%. The correlation between these variations and the control charts showed a perfect match. This proves that an excellent way to implement an air density gauge is to measure the energy absorption in a gap between an alpha or beta source and a detector.

The frequent failure issue was relieved by acknowledging the influence of a systematic error and incorporating and propagating it in their control charts. The industry standard for systematic error with regards to alpha/beta counters is 1% (1 sigma).

Equation 8 is used to propagate the errors from the sum of two random sources.

$$\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2}$$

Propagating Errors from a Sum - Equation 8

When applied to counting instruments this reduces to Equation 9 where *p* is the fractional error (0.01 in our case).

$$\sigma_{Ltd} = \sqrt{\sigma_e^2 + (\overline{x_e}p)^2}$$

Limited Deviation includes Systematic Error – Equation 9

Equation 9 limits the standard deviation of the charts to ≥1% and prevents the establishment of unreasonable limits for instrument reproducibility. The limited standard deviation method should be used when calculating control chart deviations.

Source Decay

Long term control charts may also show deviations due to source decay which may be mistaken for instrument errors. Even sources with a 20 to 30 year half life will show approximately a 1% decline after a year. This decay will be observed if raw counts are plotted over extended periods.

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One way to prevent a control chart from being skewed by source decay is to plot efficiency rather than counts; providing the source activity is decay corrected.

$$\varepsilon = \frac{x_i/t}{A_{dpm}}$$

Efficiency Factor Calculation – Equation 10

Equation 10 is used to calculate an efficiency factor where *t* is the count time in minutes and *A* is the calibrated activity at the time of the count in disintegrations per minute (dpm). This implies that the activity is decay corrected. When Protean's control chart function is used to track efficiency, decay correction is automatic assuming the source information is provided during setup.

Whenever multiplication or division is performed on two variables as in Equation 10, and each has an associated error, Equation 11 is the rule used to propagate those errors.

$$\sigma_T = x_T\sqrt{\left(\frac{\sigma_1}{x_1}\right)^2 + \left(\frac{\sigma_2}{x_2}\right)^2}$$

Propagated Errors for Multiplication or Division of Variables – Equation 11

The source manufacturer normally includes a calibration sheet specifying the error associated with the stated activity. The date of the calibration and half-life of the specific isotope is used for decay correction. Combining Equations 10 and 11 to propagate the activity error and the counting errors yields Equation 12.

$$\sigma_\varepsilon = \bar{\varepsilon}\sqrt{\left(\frac{\sigma_{Ltd}}{\bar{x}_e}\right)^2 + \left(\frac{\sigma_A}{A_{dpm}}\right)^2}$$

Deviation with Propagated Activity Error – Equation 12

Protean's computer applications that include efficiency tracking control charts use Equation 12 to compute the control chart limits. The source error provided by the manufacturer almost always dominates the combined systematic and counting error; this may suggest a valid argument for excluding it. Another alternative is to assign an error that is less conservative than that assigned by the source manufacturer – at least for control chart purposes.

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